Technical Section: Vibration and Shock

Vibration

This outline of basic vibration theory is intended to present a simplified approach to application and sizing of isolators. It will enable the design engineer to select the proper isolator to reduce the harmful effects of vibration. Obviously, real life situations are more complex than this simplified approach indicates.

Vibration is defined as a magnitude (force, displacement, or acceleration) which oscillates about a reference point. Vibration is commonly expressed in terms of frequency, cycles per second or Hertz (Hz).

Vibration problems generally fall into two classes.

1. Force excitation: The isolator is used to protect the supporting structure from forces generated by the supported mass (see Figure 1). An example is the use of motor mounts in an automobile.

2. Motion excitation: The isolator is used to protect the supported mass from disturbances of the supporting structure (see Figure 2). An example is the use of mounts under a coordinate measuring machine.

Natural Frequency is the frequency of vibration that will occur if a system is disturbed from its normal position and allowed to vibrate freely. For our purposes the natural frequency can be defined as a function of mass and stiffness or spring rate. If the spring rate is linear, the load vs. deflection curve is a straight line (Figure 3). For instance, a load of 100 pounds will cause a deflection of .20 inches. This spring will have a stiffness of:

$$K = \frac{W}{D} = \frac{100}{.20} = 500 \text{ lbs./inch}$$

Where: K = Stiffness (pounds per inch)
W = Weight of load (pounds)
D = Deflection (inches)

If we assume the supported item is a rigid body, the system will have a well-defined Natural Frequency (f_n).

$$f_n = \frac{1}{2\pi}\sqrt{\frac{Kg}{W}}$$

or, removing the constants:

$$f_n = 3.13\sqrt{\frac{K}{W}}$$

Where: W = Weight of load (pounds)
g = Acceleration due to gravity (386 in./sec.^{2})
π = 3.1416

If the frequency of the input that we are isolating from (the forcing frequency) is defined as f_f, it can be shown that if the spring has been selected so that:

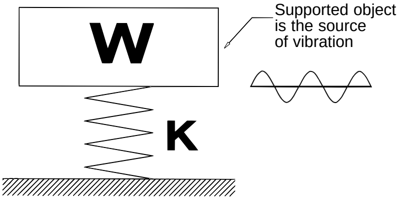

Figure 1

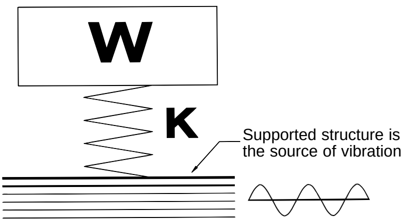

Figure 2

Load versus Deflection

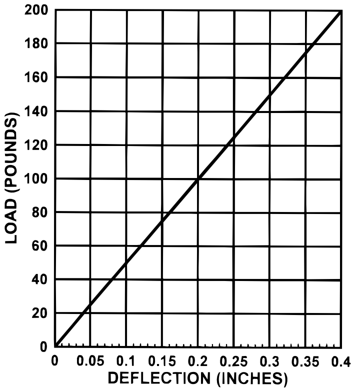

Figure 3

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$$\frac{f_f}{f_n} > \sqrt{2}$$

the displacement of the isolated item will be less than that of the input. This is the basis for vibration isolation (Figure 4).

However, if:

$$\frac{f_f}{f_n} < \sqrt{2}$$

the displacement of the isolated item will be greater than that of the input. This is the region of amplification (Figure 4).

Since Transmissibility (T) is defined as the ratio of the output to the input:

$$T = \frac{\text{output}}{\text{input}}$$

maximum transmissibility always occurs when the forcing frequency (f_f) and the natural frequency (f_n) coincide. This is commonly called the resonant point.

If T is greater than one, amplification is occurring. If T is less than one, isolation is occurring.

Figure 4 depicts typical transmissibility curves for various damping conditions.

Typical Transmissibility For Viscous Damping

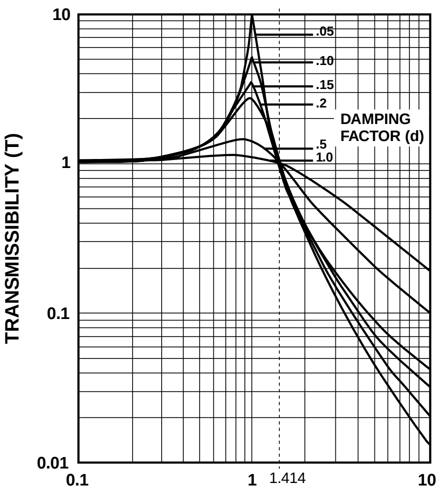

Figure 4

Damping (d) is defined as the dissipation of energy by conversion to heat. Note that damping affects the magnitude of the response; it has little affect on the frequency of the response. Figure 5 gives damping factors for some typical materials.

Typical Damping Factors

Material	d
Natural Rubber	.05
Neoprene	.05
Felt and cork	.06
Butyl	.10
Highly damped silicone	.13+
Friction damped spring	.30+

Figure 5

Figure 4 indicates that while the maximum transmissibility varies with damping, for lower damping values the crossover point is always:

$$f_n \sqrt{2}$$

The three types of damping usually encountered are friction (Coulomb), hysteretic and viscous.

Friction damping is characterized by sliding surfaces. Hysteretic damping is the damping that is inherent in a material. Viscous (or fluid) damping is characterized by proportional relationships between forces and velocities, e.g. an object moving through a liquid.

Transmissibility (T) is the ratio of the output to the input. If the input amplitude is .10 inches, and the output is .025 inches, the transmissibility will be:

$$T = \frac{\text{output}}{\text{input}} = \frac{.025}{.100} = .25$$

The percent of isolation can be expressed as:

% Isolation = (1-T) x 100

or in this case: % Isolation = (1–.25) x 100
= 75%.

Quite often the magnitude of amplification at resonance is important. This point of maximum transmissibility is solely determined by the amount of damping (d) in the isolator. For isolators, d is typically .06 to .20. A simplified expression for maximum amplification (Q) for lower damping values is given by:

$$Q = \frac{1}{2d}$$

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If d = .13 (typical of a highly damped silicone)

$$Q = \frac{1}{2(.13)} = 3.85$$

The amplification factor at resonance for most isolators varies between 2.5 and 8.0.

While damping is desirable to control the response at resonance, it actually decreases the isolation at higher frequencies. As Figure 4 indicates, the more damping in a system, the *less* isolation at frequencies above $f_n\sqrt{2}$.

If the forcing frequency (f_f) and the desired transmissibility are known, the required system natural frequency is calculated by:

$$f_n = \frac{f_f}{\sqrt{\frac{1}{T} + 1}}$$

For instance, if f_f is 20 Hz and T is .25, then the maximum acceptable f_n is 8.9 Hz.

This equation is presented in nomograph form as Figure 8.

EXAMPLE

A unit with a weight of 800 pounds is to be mounted on four isolators. The center of gravity is located at the center of the unit. The forcing frequency is 30 Hz and 80% isolation, or a transmissibility of .20 is desired.

With four isolators, the load supported by each will be 200 pounds. If the unit's center of gravity is eccentric, a load distribution analysis must be made to determine the load at each mounting point.

Loads versus natural frequency curves are available for most Tech Products isolators. Often several isolators can be selected using these curves. The load versus frequency curves for the 515 Series may result in a proper isolator selection; however, there are always other conditions to consider. These may be: shock requirements, available space, mounting orientation or environmental conditions.

First the required system natural frequency is determined:

$$f_n = \frac{f_f}{\sqrt{\frac{1}{T}+1}} = \frac{30}{\sqrt{\frac{1}{.20}+1}} = 12.2 \text{ Hz}$$

Next, choose a load versus natural frequency curve where the supported weight is about in the middle of the load range. If, after the calculations are made, desirable results are not obtained, go to the curves of the next larger or smaller mount and repeat the calculations.

Figures 6 and 7 show the curves for a typical mount that has been selected for this application. Draw a horizontal line across Figure 7 at 200 pounds on the load axis. Then draw a vertical line across Figure 7 from 12.2 Hz on the natural frequency axis. The intersection of the two lines is slightly to the left of curve -4 on Figure 7. If a vertical line is drawn to the frequency axis from the point where the 200 pound line intersects curve -4, the natural frequency value is 12.5 Hz. This is slightly higher than the 12.2 Hz calculated. However, it is close enough so that the -4 could be selected.

If f_n = 12.5 Hz is put into the transmissibility equation

$$T = \frac{1}{\left(\frac{f_f}{f_n}\right)^2 - 1}$$

T = .21 or approximately 79% isolation.

One should note that the magnitude of the input would affect the system's natural frequency. The modulus of elastomeric materials is strain sensitive, so at very small inputs the natural frequency will be slightly more than calculated and slightly less at very high inputs.

"Shortcuts"

The preceeding transmissibility equation is graphically produced in Figure 8.

Using the previous example, where the forcing frequency is 30 Hz and 80% isolation is desired: Draw a horizontal line across Figure 8 located at 30 Hz on the forcing frequency axis to the intersection of the 80% isolation line. Draw a vertical line down to the natural frequency axis. This point defines the required system's natural frequency to be approximately 12 Hz.

From the natural frequency equation given on page 6, it can be shown that the natural frequency is a function of the isolator static deflection (ΔS). That is:

$$\text{if } f_n = 3.13\sqrt{\frac{K}{W}}$$

$$\text{and } K = \frac{W}{\Delta S}$$

$$\text{then } f_n = 3.13\sqrt{\frac{1}{\Delta S}}$$

This equation is shown graphically in Figure 9.

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Load vs. Deflection (Typical Mount)

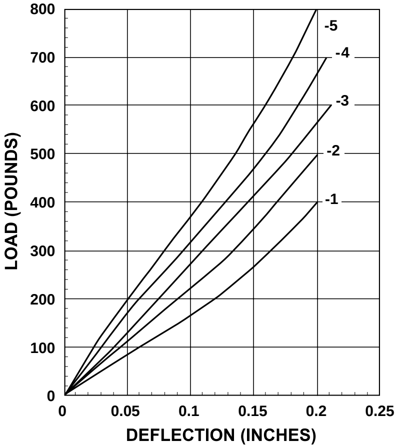

Figure 6

Load vs. Frequency (Typical Mount)

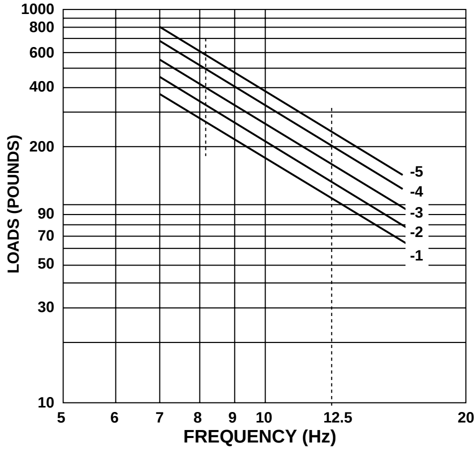

Figure 7

Vibration Mount Effectivity

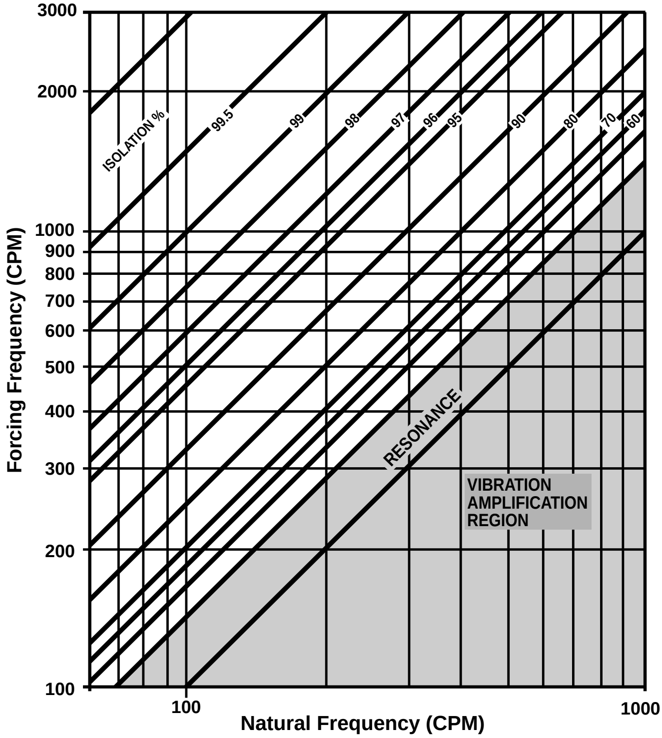

Figure 8

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If load vs. frequency curves are not available, then Figure 9 can be used to help select an isolator. The desired natural frequency is determined as in the example previously discussed (12.2 Hz). Draw a horizontal line from 12.2 Hz on the natural frequency axis to the intersection of the dark diagonal line. Draw a vertical line down to the intersection of the static deflection axis. This point, approximately .065 inches, is the static deflection required of the isolator to produce a natural frequency of 12.2 Hz.

Load deflection curves can now be used to determine what isolator will produce .065 inches deflection at the given load.

Shock

Shock is normally classified as a transient phenomenon in contrast to vibration that is normally a steady-state phenomenon.

Shock isolation is considerably different from vibration isolation. A shock isolator is an energy storage device that stores the input energy by deflecting and then releasing that energy over a longer period of time. The energy is released at the natural frequency of the shock isolation system.

Shock is normally defined by a pulse or a free-fall impact. Some typical pulse shapes are half-sine, triangular, rectangular and versed-sine.

A convenient way to analyze shock problems is to use the velocity change method. Figure 10 gives equations to calculate the velocity change (V) for various shock excitations.

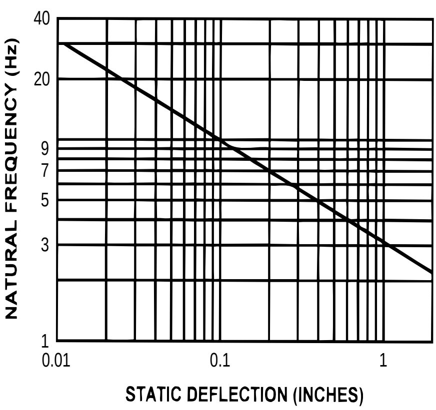


Figure 9

The transmitted shock (G_t) is given by:

$$G_t = \frac{V(2\pi f_n)}{g} = \frac{V(f_n)}{61.4}$$

The associated dynamic deflection (Δd) can be determined by:

$$\Delta d = \frac{V}{2\pi f_n}$$

EXAMPLE

A piece of equipment is subjected to a 24-inch (h) free-fall drop. It is known that the equipment cannot withstand more than 25 g's, i.e. the fragility level is 25 g's. The equipment weighs 400 pounds.

Using the transmitted shock (G_t) equation and setting G_t to 25 and solving for f_n:

$$G_t = \frac{V(f_n)}{61.4}$$

$$\text{or } f_n = \frac{G_t(61.4)}{V} = \frac{25(61.4)}{V}$$

From Figure 10, $V = \sqrt{2gh}$

where: h = drop height in inches

g = acceleration due to gravity (386 in./sec.^{2})

or $V = \sqrt{2(386)(24)} = 136$ in./sec.

The required natural frequency is:

$$f_n = \frac{(25)(61.4)}{136} = 11.3 \text{ Hz}$$

The required dynamic deflection (Δd) is:

$$\Delta d = \frac{V}{2\pi f_n} = \frac{136}{2\pi(11.3)} = 1.92 \text{ inches.}$$

Now calculate the required dynamic stiffness (K) for the system.

$$\text{Since } f_n = 3.13\sqrt{\frac{K}{W}}$$

$$K = \frac{(f_n)^2 W}{(3.13)^2} = \frac{(11.3)^2 (400)}{(3.13)^2}$$

or K = 5213 lbs./inch

We have now found that to protect the equipment from the 24-inch drop we need:

1. A system's natural frequency of 11.3 Hz.
2. A dynamic deflection of 1.92 inches.
3. A dynamic system stiffness of 5213 lbs./inch.

All three of these conditions must be met to assure that no more than 25 g's is transmitted to the equipment.

Vibration and Shock

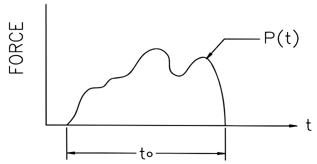
Note that the dynamic stiffness (K) found is the *system* stiffness. It must be divided by the number of mounts to determine the stiffness required per mount.

If both vibration and shock are present, both must be considered. Quite often the final solution is a compromise.

Shock spectra nomographs, which can greatly simplify the preceding selection process, are available at no cost from Tech Products Corporation.

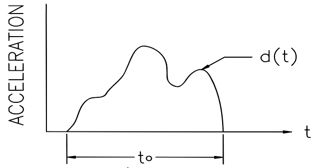
For more complicated, eccentrically loaded systems, Tech Products will provide free analytical assistance.

Typical Shock Excitations



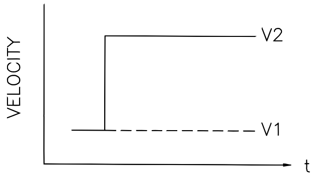
$$V = \frac{1}{m}\int_0^{t_o} d(t)dt$$

Force Impulse



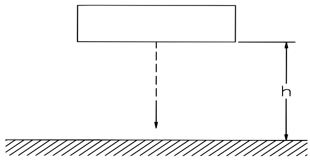
$$V = \int_0^{t_o} d(t)dt$$

Acceleration Impulse



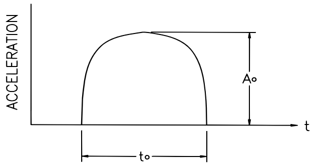
$$V = V_2 - V_1$$

Velocity Shock



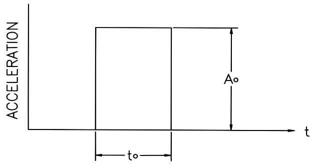
$$V = \sqrt{2gh} \text{ (inelastic impact)}$$
$$V = 2\sqrt{2gh} \text{ (elastic impact)}$$

Free-Fall Impact



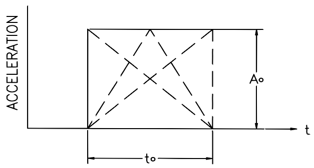
$$V = \frac{2g}{\pi} A_o t_o$$

Half-Sine Acceleration



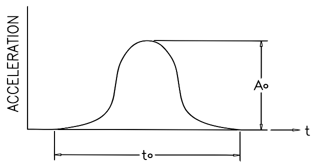
$$V = gA_o t_o$$

Rectangular Acceleration



$$V = \frac{g}{2} A_o t_o$$

Triangular Acceleration



$$V = \frac{g}{2} A_o t_o$$

Versed-Sine Acceleration

Figure 10